

# RADIATIVE CORRECTIONS TO W-PAIR MEDIATED FOUR-FERMION PRODUCTION AT LEP2

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We present the radiative corrections to off-shell  $W$ -pair production, as calculated within the double-pole approximation.

## 1 Introduction

This report summarizes the results on electroweak radiative corrections to reactions that involve the production and subsequent decay of pairs of  $W$  bosons, which are of prime interest at LEP2. Unless stated otherwise we use the reaction

$$e^+(q_1) e^-(q_2) \rightarrow \mu^+(k_1) \nu_\mu(k'_1) \tau^-(k_2) \bar{\nu}_\tau(k'_2) \quad (1)$$

as typical example, but all four-fermion final states originating from  $W$ -boson pairs can be treated in a similar way. Since the RADCOR 98 conference took place our results have been finalized and discussed elsewhere<sup>1</sup>. A highlight of the main points is given here. For related issues, see these proceedings<sup>2</sup>.

A complete calculation of  $\mathcal{O}(\alpha)$  radiative corrections to a process with six external particles is beyond present possibilities. However, the possible presence of two resonances as intermediate state in reaction (1) offers an additional expansion parameter  $\Gamma_W/M_W$  besides  $\alpha$ . Since the ratio between the width and mass of the  $W$  boson is also of order  $\alpha$ , a double expansion both in  $\alpha$  and  $\Gamma_W/M_W$  is a natural and economic way to simplify the calculation. Thus the aim is to calculate radiative corrections of  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\Gamma_W/M_W)$ , while neglecting higher-order terms such as  $\alpha\Gamma_W/M_W$ . The way in which the expansion is carried out is called the double-pole approximation<sup>3</sup> (DPA), which possesses the important feature of maintaining gauge invariance.

## 2 The Born cross-section in the double-pole approximation

As an illustration, we shall first apply the DPA method to the Born cross-section. When one calculates the Born cross-section  $d\sigma^0$  in the usual way, gauge invariance demands not only the inclusion of all diagrams but also a

gauge-invariant procedure to incorporate the width in the  $W$ -boson propagators. For a discussion of these issues we refer to the literature<sup>4,5</sup>. Numerically the three diagrams involving two resonant  $W$ -boson propagators (called CC03) are the dominant ones. The diagrams containing a single resonant  $W$  boson or no resonant  $W$  boson at all are suppressed by  $\Gamma_W/M_W$  and  $(\Gamma_W/M_W)^2$ , respectively, although the exchange of almost real photons may enhance suppressed terms. The CC03 part of the cross-section is not gauge-invariant, but its residue in the DPA is. Therefore we split up the lowest-order cross-section according to

$$d\sigma^0 = d\sigma_{\text{DPA}}^0 + (d\sigma^0 - d\sigma_{\text{DPA}}^0), \quad (2)$$

where the first term originates from the CC03 amplitude in the DPA limit and the second term is suppressed by at least  $\Gamma_W/M_W$ .

The way in which  $d\sigma_{\text{DPA}}^0$  is defined starts with the CC03 amplitude

$$\mathcal{M} = \sum_{\lambda_1, \lambda_2} \Pi_{\lambda_1 \lambda_2}(M_1, M_2) \frac{\Delta_{\lambda_1}^{(+)}(M_1)}{D_1} \frac{\Delta_{\lambda_2}^{(-)}(M_2)}{D_2}, \quad (3)$$

where the summation runs over the  $W^\pm$  helicities  $(\lambda_{1,2})$ . The quantities  $\Delta^{(\pm)}$  and  $\Pi$  are the off-shell amplitudes for  $W^\pm$  decay and  $W$ -pair production, respectively. The inverse propagators  $D_i$  ( $i = 1, 2$ ) are defined as

$$D_i = M_i^2 - M_W^2 + i\Gamma_W M_W, \quad M_i^2 = (k_i + k'_i)^2. \quad (4)$$

The momenta of the resonant  $W^\pm$  bosons will be indicated by  $p_{1,2}$ . Introducing the production/decay phase-space factors

$$d\Gamma_{\text{pr}} = \frac{1}{(2\pi)^2} \delta(q_1 + q_2 - p_1 - p_2) \frac{d\vec{p}_1}{2p_{10}} \frac{d\vec{p}_2}{2p_{20}}, \quad (5)$$

$$d\Gamma_{\text{dec}}^+ = \frac{1}{(2\pi)^2} \delta(p_1 - k_1 - k'_1) \frac{d\vec{k}_1}{2k_{10}} \frac{d\vec{k}'_1}{2k'_{10}}, \quad (6)$$

and a similar expression for  $d\Gamma_{\text{dec}}^-$ , the differential cross-section reads

$$\begin{aligned} d\sigma = & \frac{1}{2s} \sum_{\lambda_1, \lambda_2, \lambda'_1, \lambda'_2} \mathcal{P}_{[\lambda_1 \lambda_2][\lambda'_1 \lambda'_2]}(M_1, M_2) d\Gamma_{\text{pr}} \times \mathcal{D}_{\lambda_1 \lambda'_1}(M_1) d\Gamma_{\text{dec}}^+ \times \\ & \times \mathcal{D}_{\lambda_2 \lambda'_2}(M_2) d\Gamma_{\text{dec}}^- \times \frac{1}{2\pi} \frac{dM_1^2}{|D_1|^2} \times \frac{1}{2\pi} \frac{dM_2^2}{|D_2|^2}, \end{aligned} \quad (7)$$

where

$$\mathcal{P}_{[\lambda_1 \lambda_2][\lambda'_1 \lambda'_2]}(M_1, M_2) = \sum_{e^\pm \text{ helicities}} \Pi_{\lambda_1 \lambda_2}(M_1, M_2) \Pi_{\lambda'_1 \lambda'_2}^*(M_1, M_2), \quad (8)$$

$$\mathcal{D}_{\lambda_i \lambda'_i}(M_i) = \sum_{\text{fermion helicities}} \Delta_{\lambda_i}(M_i) \Delta_{\lambda'_i}^*(M_i). \quad (9)$$

For the double-pole approximation of Eq. (7) we choose the prescription  $p_i^2 = M_i^2 \rightarrow M_W^2$  both in the phase-space factors and in  $\mathcal{P}$  and  $\mathcal{D}$ , which can be identified as on-shell density matrices. Strictly speaking, the double-pole residue should be taken at the complex pole,  $M_i^2 \rightarrow M_W^2 - i\Gamma_W M_W$ . In order to avoid problems with complex kinematics and complicated analytic continuations, however, we use the limit  $M_i^2 \rightarrow M_W^2$  instead. In our procedure the phase-space factors are given by the on-shell expressions in the laboratory system, with the production/decay angles being free and the energies being fixed by the DPA limit. This is achieved by writing  $d\sigma$  as a differential in both  $M_i^2$  and the solid production/decay angles. Distributions in  $M_i^2$  are solely determined by  $dM_i^2/|D_i|^2$  and not anymore by the matrix element, at least not at Born level. The full integration over the  $M_i^2$  distribution is approximated by

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} dM_i^2 \frac{1}{|D_i|^2} = \frac{1}{2M_W \Gamma_W}. \quad (10)$$

For distributions in other variables, like energies, an appropriate off-shell Jacobian should be inserted.

In the following we shall consider the radiative corrections to  $d\sigma_{\text{DPA}}^0$ , so that the  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\Gamma_W/M_W)$  corrected cross-section is obtained as

$$d\sigma = d\sigma_{\text{DPA}}^0 (1 + \delta_{\text{DPA}}) + (d\sigma^0 - d\sigma_{\text{DPA}}^0). \quad (11)$$

### 3 Radiative corrections in the double-pole approximation

Besides the usual distinction between virtual and real corrections one should now also distinguish between factorizable and non-factorizable corrections. The factorizable corrections are corrections that apply to the production or decay parts separately, whereas the non-factorizable corrections can be regarded as interference effects between different stages of the off-shell process.

The factorizable corrections comprise the on-shell corrections to the  $W$ -pair production process and the on-shell corrections to the partial decay widths of the  $W$  bosons, generalized to the density matrices  $\mathcal{P}$  and  $\mathcal{D}$ . We have calculated these generalized corrections by extending existing calculations for  $W$ -pair production<sup>6</sup> and decay<sup>7</sup>. For virtual corrections and soft-photon radiation [ $E_\gamma \ll \Gamma_W$ ] no further clarifications are required.

The radiation of more energetic photons, however, requires some special care in the treatment of both the matrix element and the phase space. When a

photon with momentum  $k$  is emitted from one of the intermediate  $W$  bosons, the resulting product of  $W$ -boson propagators can be written as

$$\frac{1}{D_i D_{i\gamma}} = \frac{1}{2kp_i} \left[ \frac{1}{D_i} - \frac{1}{D_{i\gamma}} \right], \quad (12)$$

where

$$D_{i(i\gamma)} = M_{i(i\gamma)}^2 - M_W^2 + i\Gamma_W M_W, \\ M_i^2 = (k_i + k'_i)^2, \quad M_{i\gamma}^2 = (k_i + k'_i + k)^2, \quad p_i = k_i + k'_i (+k). \quad (13)$$

The first term on the r.h.s of Eq. (12) can now be interpreted as belonging to the production of a  $W$  boson plus a photon, whereas the second term corresponds to the radiative decay of the  $W$  boson. Therefore, the real-photon bremsstrahlung cross-section has the following structure

$$d\sigma = \frac{1}{2s} |\mathcal{M}_\gamma|^2 d\Gamma_{4f\gamma} = \frac{1}{2s} |\mathcal{M}_0 + \mathcal{M}_+ + \mathcal{M}_-|^2 d\Gamma_{4f\gamma}, \quad (14)$$

with  $d\Gamma_{4f\gamma}$  indicating the complete off-shell phase-space factor. The exact amplitude in Eq. (14) has been divided into parts related to radiative  $W$ -pair production ( $\mathcal{M}_0$ ) and radiative  $W$ -boson decays ( $\mathcal{M}_\pm$ ).

For hard photons [ $E_\gamma \gg \Gamma_W$ ] the interference terms between these different radiation sources are suppressed, since the propagators peak in different parts of phase space. Thus merely the quadratic  $|\mathcal{M}_{0,\pm}|^2$  parts need to be retained in Eq. (14). Introducing the radiative production/decay phase-space factors

$$d\Gamma_{\text{pr}}^\gamma = \frac{1}{(2\pi)^2} \delta(q_1 + q_2 - p_1 - p_2 - k) \frac{d\vec{p}_1}{2p_{10}} \frac{d\vec{p}_2}{2p_{20}} \frac{d\vec{k}}{(2\pi)^3 2k_0}, \quad (15)$$

$$d\Gamma_{\text{dec}}^{+\gamma} = \frac{1}{(2\pi)^2} \delta(p_1 - k_1 - k'_1 - k) \frac{d\vec{k}_1}{2k_{10}} \frac{d\vec{k}'_1}{2k'_{10}} \frac{d\vec{k}}{(2\pi)^3 2k_0}, \quad (16)$$

and a similar expression for  $d\Gamma_{\text{dec}}^{-\gamma}$ , the complete off-shell phase-space factor  $d\Gamma_{4f\gamma}$  can be written in three equivalent forms

$$d\Gamma_{4f\gamma} = d\Gamma_0^\gamma = d\Gamma_{\text{pr}}^\gamma \cdot d\Gamma_{\text{dec}}^+ \cdot d\Gamma_{\text{dec}}^- \cdot \frac{dM_1^2}{2\pi} \cdot \frac{dM_2^2}{2\pi}, \quad (17)$$

$$d\Gamma_{4f\gamma} = d\Gamma_+^\gamma = d\Gamma_{\text{pr}} \cdot d\Gamma_{\text{dec}}^{+\gamma} \cdot d\Gamma_{\text{dec}}^- \cdot \frac{dM_{1\gamma}^2}{2\pi} \cdot \frac{dM_2^2}{2\pi}, \quad (18)$$

and a similar expression for  $d\Gamma_-^\gamma$ . Consequently the hard-photon cross-section can be rewritten as

$$d\sigma = \frac{1}{2s} \left[ |\mathcal{M}_0|^2 d\Gamma_0^\gamma + |\mathcal{M}_+|^2 d\Gamma_+^\gamma + |\mathcal{M}_-|^2 d\Gamma_-^\gamma \right]. \quad (19)$$

When the appropriate DPA limits are taken, three distinct factorizable contributions emerge, with the photon assigned to either the production stage or one of the decay stages.

For semi-soft photons [ $E_\gamma = \mathcal{O}(\Gamma_W)$ ] the matrix element can be written in a factorized soft-photon form with the propagators  $D_i$  and  $D_{i\gamma}$  kept exact. This has two consequences. One is that in this regime  $|\mathcal{M}_\pm|^2$  can be replaced by simplified expressions, which are useful for discussing final-state radiation (FSR) effects on the resonance shapes<sup>1,8</sup>. The second consequence concerns the non-factorizable interference terms in Eq. (14) between the different radiative production and decay stages. These interference terms cannot be neglected anymore in the semi-soft regime, as the propagators  $D_i$  and  $D_{i\gamma}$  start to overlap. In a similar way, also the virtual non-factorizable corrections originate solely from semi-soft photonic interconnection effects between the different production and decay stages. For detailed discussions of the non-factorizable corrections we refer to the literature<sup>9,10,11</sup>. We merely state here that their numerical impact is in general relatively small.

## 4 Numerical results

For the numerical evaluations we adopt the LEP2 input-parameter scheme<sup>4</sup>. The independent input parameters are the Fermi constant  $G_\mu$ , the fine-structure constant  $\alpha$ , the masses of the light fermions (which reproduce the experimentally determined hadronic vacuum polarization), and the masses of the  $W, Z$  bosons. Subsequently, we make use of the Standard Model prediction for  $G_\mu$  in terms of the above parameters, the top-quark mass  $m_t$ , the Higgs-boson mass  $M_H$ , and the strong coupling  $\alpha_S$ . In this way  $m_t$  will be fixed once the other parameters have been chosen. In the lowest-order cross-sections we use  $G_\mu$  instead of  $\alpha$  and compensate for this replacement in the one-loop corrections, i.e. we use the so-called  $G_\mu$  parametrization.

In the following we present a small selection of interesting numerical results as obtained with our DPA procedure<sup>1</sup>.

In Fig. 1 we start off with the DPA energy spectrum of the photon ( $d\sigma/dE_\gamma$ ) and the separate contributions to it from the production stage, the decay stages, and the semi-soft interference terms. As can be seen clearly, the contribution from the  $W^+$  decay comes out larger than the one from the  $W^-$

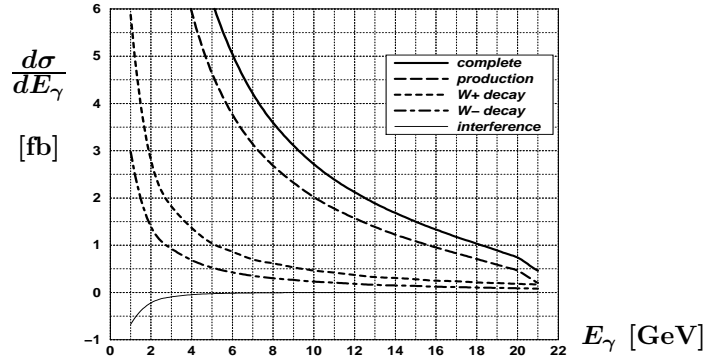


Figure 1: The photon-energy distribution  $d\sigma/dE_\gamma$  for the  $\mu^+\nu_\mu\tau^-\bar{\nu}_\tau\gamma$  final state at the LEP2 energy of 184 GeV. In addition the separate production and decay contributions are given.

decay. This is a result of the fact that we consider reaction (1) with its specific choice of charged leptons. It simply means that the photon-energy spectrum originating from the decay  $W \rightarrow \mu\nu_\mu\gamma$  exceeds the one from  $W \rightarrow \tau\nu_\tau\gamma$ , as expected from collinear-photon considerations.

For inclusive photons, the corrections to the total cross-section and the  $(W^\pm)$  production-angle distribution are essentially the same as for stable  $W$  bosons, provided one treats the widths in the propagators and in the decay channels in the same way. As is well known, these corrections are dominated by the large logarithms of initial-state radiation (ISR). If one does not integrate over the invariant masses  $M_1$  and/or  $M_2$ , one will in addition observe FSR and non-factorizable effects. The latter effects are in general relatively small (see e.g. Fig. 1). The FSR effects, however, may lead to a sizeable distortion of the invariant-mass distributions<sup>8</sup>.

The FSR distortion effects are displayed in Figs. 2 and 3 for the invariant-mass distribution  $d\sigma/dM_1^2$  of reaction (1), which involves the decay  $W^+ \rightarrow \mu^+\nu_\mu$ . The effect decreases for the decay  $W^+ \rightarrow \tau^+\nu_\tau$  and increases for  $W^+ \rightarrow e^+\nu_e$ . The corresponding shift in the peak position of the Breit-Wigner line shape is found to be  $-20$ ,  $-39$ , and  $-77$  MeV for the decays  $W^+ \rightarrow \tau^+\nu_\tau$ ,  $\mu^+\nu_\mu$ , and  $e^+\nu_e$ , respectively. These large distortion effects hinge on the fact that one is able to determine  $M_1$  for a dilepton final state. In practical experimental situations the effect will be smaller, for instance because a photon emitted collinear with the charged lepton may not be separately detectable. Nevertheless it is useful to be aware of this effect.

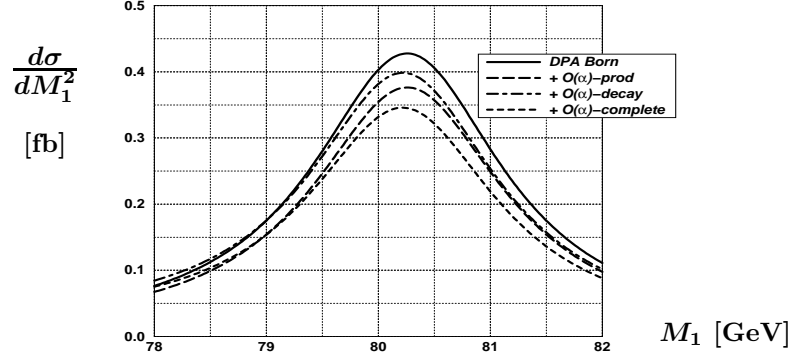


Figure 2: The invariant-mass distribution  $d\sigma/dM_1^2$  for the  $\mu^+\nu_\mu\tau^-\bar{\nu}_\tau$  final state at the LEP2 energy of 184 GeV.

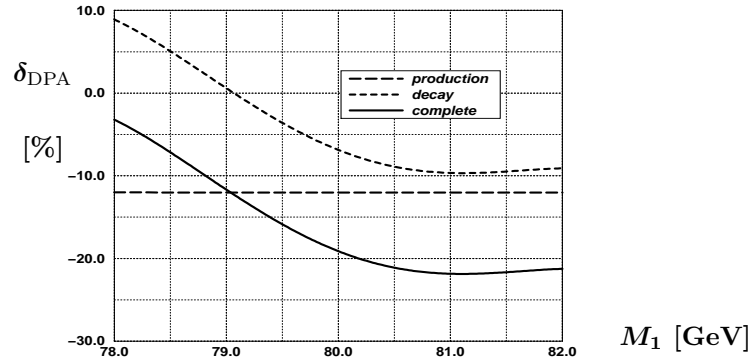


Figure 3: The relative correction factors  $\delta_{\text{DPA}}$  corresponding to Fig. 2.

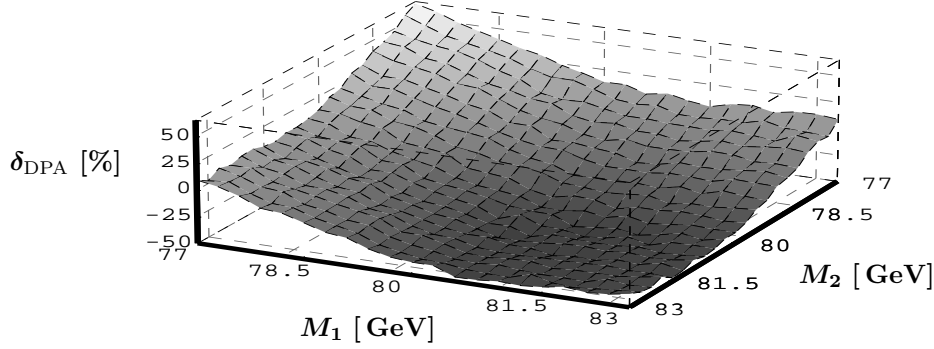


Figure 4: Correction to the double invariant-mass distribution  $d\sigma/dM_1^2 dM_2^2$  for the  $e^+\nu_e e^-\bar{\nu}_e$  final state at the LEP2 energy of 184 GeV.

$\delta_{\text{dec}}^+(M_1)$				$\delta_{\text{nf}}(M_1, M_2)$			
$\Delta_1$	decay channel			$\Delta_1$	$\Delta_2$		
	$e^+\nu_e$	$\mu^+\nu_\mu$	$\tau^+\nu_\tau$		-1/2	0	1/2
-1/2	-1.4	-0.8	-0.5	-1/2	+0.5	+0.2	-0.1
0	-15.0	-7.8	-4.0	0	+0.2	+0.0	-0.2
1/2	-17.3	-9.0	-4.6	1/2	-0.1	-0.2	-0.4

Table 1: Relative correction factors [in %] for the double invariant-mass distribution  $d\sigma/dM_1^2 dM_2^2$  at the LEP2 energy of 184 GeV. Left: the corrections from the  $W^+$ -boson decay stage  $\delta_{\text{dec}}^+(M_1)$  for different leptonic decay channels. Right: the non-factorizable corrections  $\delta_{\text{nf}}(M_1, M_2)$ . Three near-resonant invariant masses are considered:  $\Delta_i = (M_i - M_W)/\Gamma_W = -1/2, 0, 1/2$ .

For a double invariant-mass distribution  $d\sigma/dM_1^2 dM_2^2$  the distortion becomes even more pronounced, as can be seen from Fig. 4 where the  $e^+\nu_e e^-\bar{\nu}_e$  final state is considered. Explicit numbers can be found in Table 1, where we have split up the relative correction factor into production, decay, and non-factorizable contributions according to

$$\delta_{\text{DPA}}(M_1, M_2) = \delta_{\text{pr}} + \delta_{\text{dec}}^+(M_1) + \delta_{\text{dec}}^-(M_2) + \delta_{\text{nf}}(M_1, M_2).$$

Here the correction  $\delta_{\text{pr}}$  from the production part is independent of  $M_i$  and equals  $-12.0\%$  at the considered energy of 184 GeV.



## 5 Conclusions

We have summarized the gauge-invariant DPA calculation of the  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\Gamma_W/M_W)$  radiative corrections to  $W$ -pair mediated four-fermion production. Special attention has been paid to a practical implementation of the DPA scheme and to the definition of the so-called factorizable and non-factorizable corrections. This DPA procedure can also be used for other reactions where unstable particles are produced in pairs.

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